* Calcul de $\int_{\lambda}^{\alpha} Log(1+\frac{1}{x})dx$ faisons une intégration par parties

Posons $\begin{cases} u'(x)=1 \\ v(x)=Log(1+\frac{1}{x}) \end{cases}$ et $\begin{cases} u'(x)=x \\ v(x)=-\frac{1}{x(1+x)} \end{cases}$

d'où $\int_{\lambda}^{\alpha} Log(1+\frac{1}{x})dx = \left[xLog(1+\frac{1}{x})\right]_{\lambda}^{\alpha} + \int_{\lambda}^{\alpha} \frac{1}{1+x}dx$

$= \alpha Log(1+\frac{1}{\alpha}) - \lambda Log(1+\frac{1}{\lambda}) + \left[Log(1+x)\right]_{\lambda}^{\alpha}$

$= \alpha Log(1+\frac{1}{\alpha}) - \lambda Log(1+\frac{1}{\lambda}) + Log(\frac{1+\alpha}{1+\lambda})$

* Calcul de $\int_{\lambda}^{\alpha} (\frac{1}{4x^2}-\frac{1}{x})dx$

$\int_{\lambda}^{\alpha} (\frac{1}{4x^2}-\frac{1}{x})dx = \left[-\frac{1}{4x}-Logx\right]_{\lambda}^{\alpha} = Log(\frac{\lambda}{\alpha}) + \frac{1}{4\lambda} - \frac{1}{4\alpha}$ on obtient

donc

$\int_{\lambda}^{\alpha} f(x)dx = \alpha Log(1+\frac{1}{\alpha}) - \lambda Log(1+\frac{1}{\lambda}) + Log(\frac{1+\alpha}{1+\lambda}) + Log(\frac{\lambda}{\alpha}) + \frac{1}{4\lambda} - \frac{1}{4\alpha}$

$= \alpha Log(1+\frac{1}{\alpha}) - \lambda Log(1+\frac{1}{\lambda}) + Log(\frac{1+\alpha}{\alpha}) - Log(\frac{1+\lambda}{\lambda}) + \frac{1}{4\lambda} - \frac{1}{4\alpha}$

$= (\alpha+1)Log(1+\frac{1}{\alpha}) - (\lambda+1)Log(1+\frac{1}{\lambda}) + \frac{1}{4\lambda} - \frac{1}{4\alpha}$

Conclusion : $A(\lambda) = (\alpha+1)Log(1+\frac{1}{\alpha}) - (\lambda+1)Log(1+\frac{1}{\lambda}) + \frac{1}{4\lambda} - \frac{1}{4\alpha}$

U.A

b) $\lim_{\lambda\to+\infty} (\lambda+1)Log(1+\frac{1}{\lambda}) = \lim_{\lambda\to+\infty} \frac{(\lambda+1)}{\lambda} Log \frac{(1+\frac{1}{\lambda})}{\frac{1}{\lambda}}$

or $\lim_{\lambda\to+\infty} \frac{(\lambda+1)}{\lambda} = 1$ et $\lim_{\lambda\to+\infty} Log \frac{(1+\frac{1}{\lambda})}{\frac{1}{\lambda}} = \lim_{X\to 0} \frac{Log(1+X)}{X} = 1$